EXPERIMENTAL OBSERVATIONS ON FLOW PATTERNS AND ENERGY LOSSES FOR OSCILLATORY FLOW IN DUCTS CONTAINING SHARP EDGES

Department of Chemical Engineering, University of Cambridge, Pembroke Street, Cambridge CB2 3RA, U.K.

(Received 23 June 1988; accepted 18 October 1988)

Abstract—We report experimental observations on both flow patterns and energy losses for oscillatory flow in geometries that can contain sharp edges. A 'U' shaped manometer arrangement is chosen with either a rectangular or cylindrical cross section. Sharp edges are introduced either as right-angled bends or baffled inserts and fluid oscillation occurs as a periodically decaying amplitude from a preset starting height. Flow visualization observations show clearly that large scale eddy mixing can be generated in regions between sharp edges. Information from the measured surface height decay enables associated viscous and eddy dissipation energy losses to be determined.

INTRODUCTION

In this paper we wish to provide clear experimental observations showing that large scale and efficient eddy mixing can be readily achieved in ducts that contain sharp edges and where the flow is oscillatory. The effect is thought to have potential within the field of chemical engineering in terms of providing a mechanism for mixing, enhanced radial transport, improved surface purging properties and also providing a route towards obtaining a near plug flow residence time distribution within a baffled tube.

With the notable exception of Bellhouse et al. (1973) little attention appears to have been paid towards exploiting oscillatory flow in ducted geometries. Bellhouse et al. used oscillatory flow in high voidage, furrowed channels in order to produce eddy motions that gave enhanced blood oxygenation. The flow geometry used for the device and its associated flow patterns were numerically modelled and observed by Sobey (1980) and Stephanoff et al. (1980). They characterized their flows in terms of two parameters, namely the peak Reynolds number, \( Re_o = \frac{U_p H}{v} \), and the Strouhal number, \( St = \frac{\omega H}{4\pi U_o} = \frac{H}{4\pi x_o} \), where \( U_o \) is the maximum velocity at the maximum channel width \( H \), \( v \) is the kinematic viscosity and \( \omega \) the angular frequency of oscillation with centre to peak amplitude \( x_o \). In general Bellhouse worked in the range \( Re_o = 0-100 \), and \( St = 0.5 \times 10^{-2} \).

Enhanced eddy mixing using oscillatory flow is also achieved in a low voidage geometry by using pulsed packed columns as described, for example, by Goebel et al. (1986). In terms of ducted flows little work has been carried outChan and Baird (1974) carried out studies of oscillatory flow in pipes containing no sharp edges and compared their results with predictions from solutions of the Navier-Stokes equation. A number of studies have been conducted on energy losses in freely oscillating manometer columns (Biery, 1969), however again all of these observations were carried out with smooth walled tubes.

The essential requirements to obtain eddy mixing is to have a reversing flow around an edge which is sharp enough to cause flow separation. Keulegan and Carpenter (1958) studied the effect of flow oscillation on cylinders and plates and established that the separation depends on the Keulegan-Carpenter number \( K = \frac{U_p T}{D} \), \( T \) is the period of oscillation and \( D \) the cylinder diameter. This can be rewritten as \( K = \frac{\pi x_o}{r} \), where \( r \) is the radius of cylinder or more generally the radius of curvature. Separation was observed to occur generally when \( K \approx > 3 \).

Knott and Mackley (1980) observed that oscillatory flow around sharp edges could give rise to complex eddy motions and it is the related effect in ducted flow that we wish to demonstrate in this paper. In order to encourage separation, the work of both Keulegan and Carpenter (1958) and Knott and Mackley (1980) suggest that sharp edges or edges with a small radius of curvature are preferred. No clear criteria appears in the literature as to the Reynolds number requirement or geometrical conditions that will give efficient mixing.

EXPERIMENTAL GEOMETRIES AND PROCEDURE

Two experimental geometries were examined. Essentially two-dimensional flows were examined using a rectangular sectioned geometry shown in Fig. 1 and flows in a cylindrical section geometry were studied in an apparatus shown in Fig. 2. Both systems are U-shaped ducts and in the case of Fig. 1, right-angled bends can systematically be introduced using the set of

1To whom correspondence should be addressed.
Fig. 1. Schematic diagram of two-dimensional U-shaped duct showing four different centre sections.

Fig. 2. Schematic diagram of cylindrical section duct. Baffle design shown as insert (dimensions in mm).
inserts shown in the figure. In the case of the cylindrical duct, sharp edges are systematically introduced by adding baffles of the dimensions shown in the figure into the bottom section of the tube. In both apparatus, the fluid in one limb of the U-tube is initially raised to a starting height $x_0$ above the equilibrium datum level by application of a partial vacuum. Fluid is held at that position with “cling film” encapsulating one limb. An experiment is conducted by quickly removing the cling film and observing the natural decay of oscillations within the duct.

The apparatus is made from Perspex and this enables photographic observations to be made. Planar illumination using a mercury vapour lamp

---

Fig. 3. Photograph of flow pattern observed at the bottom of the outer limbs. $x_0 = 7$ cm. Flow at maximum velocity on first downward oscillation (two-dimensional duct).

Fig. 4. Photograph of flow pattern observed with the centre straight section. $x_0 = 7$ cm. Flow at maximum velocity on first downward oscillation (two-dimensional duct).
Fig. 5. Photographs of the time evolution of eddies observed for the single right-angled bend section, $s_z = 11$ cm. (a) Primary flow L-R, start of oscillation, (b) primary flow L-R, approaching first flow reversal, (c) at first flow reversal, (d) primary flow R-L.
was projected across the section of interest and photographs were taken when viewed at right angles to the illuminated beam. Streak lines were detected by seeding the water with a dispersion of fine polyethylene particles. In the case of the cylindrical apparatus a flat faced sighting box was fitted around the outside of the tube in order to reduce optical distortion at the curved air perspex interface.

We are mainly interested in events occurring within the central duct and in designing the outer limbs of the U-tubes, the inside edges of the ducts were radiusussed as shown in both Figs 1 and 2. It was found for all starting amplitudes tested no separation of the flow occurred in the outer limbs and the damping in the outer limbs alone was observed to be linear.

Energy losses in the system were observed in a similar way to that previously reported by Knott and Mackley (1980) using a “Churchill Control” wave monitor to measure surface height decay. The device consists of a twin wire probe placed in one limb of the manometer and enables surface heights to be measured accurately with a time resolution of about 0.01 s. The calibrated voltage, measuring surface height was recorded using a PDP11 computer. Data points were recorded every 0.04 s for a duration of 20 s. Typically starting amplitudes ranged from 4-15 cm and the frequency of oscillation was of order 0.7 Hz. All experiments were conducted using tap water at room temperature.

**EXPERIMENTAL OBSERVATIONS**

**Two-dimensional flow geometry**

**Flow patterns.** Figures 3–9 are photographs and diagrams showing the observed flow patterns in different duct sections. Figure 3 is a photograph of the streak pattern at the base of the outer limbs of the duct. For all of the starting amplitudes examined namely 4, 7, 11 and 15 cm the flow was similar to that shown in the figure; no eddies or secondary flows being detectable. Figure 4 is a photograph of the straight centre section duct. The photograph shows what appear to be parallel streak lines. This was observed for all amplitudes of oscillation and at all stages during the oscillation.

Figure 5 shows the time evolution of the eddies for a single right-angled bend. By observing several photographic sequences, we have constructed a schematic sequence of eddy formation, which is shown in Fig. 6. The exact details of the flow will depend to a certain extent on the starting amplitude of the oscillation. However, the general effect of separation, followed by the formation of eddy pairs was seen for all starting amplitudes above about 20 mm. With the initiation of flow, separation occurs downstream from the inside sharp edge of the duct and a bound vortex region is formed. As the flow reaches its first minimum and reverses in direction; the flow channels between the inner wall and the region of the bound vortex; convecting the bound vortex away from the wall and into the main stream flow. With flow reversal a new bound vortex forms in the new downstream position from the edge. Each flow reversal causes a further eddy to be formed, which is subsequently ejected into the bulk of the flow on flow reversal. Eddies that have been ejected into the flow interact in a complex way which was difficult to follow. The eddy pair formation and ejection from the downstream wall has strong similarities with the mechanism of eddy formation reported in Knott and Mackley (1980) for oscillatory flows past flat plates and into parallel sided cylinders.

The formation of eddies in the double right-angled bend section is shown photographically in Fig. 7 and schematically in Fig. 8. Separation is again seen downstream from each edge. With reversal of flow, both bound vortices are brought away from the walls and then convected with the reversing flow, generally into the stagnant zones in the outer corners of the duct. From the later pictures in the sequence it was seen that very effective mixing of the flow is apparently achieved. The final photographic sequence shown in Fig. 9 is for the four right-angle bend section. These photographs show the same mechanism for eddy formation and clearly indicate that macroscopically well-mixed flows rapidly develop as the eddies formed at each sharp edge interact with one another.

**Energy losses.** The time evolution of surface height decay is shown in Fig. 10. The full periodic curve is
Fig. 7 Photographs of the time evaluation of eddies observed for the two right angled bend section. $x = 11$ cm. (a) Primary for L-R, start of oscillation, (b) primary flow L-R, approaching first flow reversal, (c) at first flow reversal, (d) primary flow L-R.
Oscillatory flow in ducts containing sharp edges

Fig. 8. Schematic diagram showing the evolution of eddies for a two right-angled bend section.

shown for the case of four right-angled bends and the decay envelope for the other three duct sections. In each case the starting amplitude was 11 cm and the decay envelopes show clearly that dissipation progressively increases as the number of right-angled bends are introduced into the system.

Tubular flow geometry

Flow patterns. To establish the flow patterns in the apparatus, several sequences of slow motion cine film were made in addition to 35 mm time sequence photographs. We report below our findings obtained from the visual, cine and photographic observations taken under different test conditions.

No baffles

Moderate starting amplitude (5–15 cm). The fluid is seen to move in plug flow with a laminar boundary layer approximately 3 mm thick moving almost 90° out of phase with the bulk flow. This latter effect is particularly noticeable when the flow reverses direction, as at this point the direction of particle motion within the boundary layer is opposite to that of the particle motion in the core of the tube. Our observation of an out-of-phase 3 mm boundary layer is consistent with the predictions of the solution for the Navier–Stokes equation applied to oscillatory boundary conditions in a tube (see, for example, Schlichting, 1979). For the situation described, no longitudinal or radial dispersion was observed as would be expected for this flow regime.

High starting amplitudes (45 cm. Note energy loss measurements were not made at this amplitude.) It would appear that the fluid motion was initially turbulent for the first and possibly second oscillation as might be expected for such a system with a peak Reynolds number of 60,000 based on tube diameter. The transition from laminar to turbulent flow in a tube without baffles is not easily identified with the tracer particle flow visualization technique. For turbulent flow in a tube the particle trajectory is, in general, only a few degrees different from the laminar flow trajectory parallel to the tube wall and hence the overall optical effect in the two flow regimes is similar. There is however an interesting but slight difference in the texture of the observed flow pattern and this can be used to detect the transition. In addition, in the turbulent regime the presence of the boundary layer is not clearly seen.

The Valensi number \( V_a = \omega R^2/\nu \) is of importance in oscillatory flows, where \( \omega \) is the angular frequency of oscillation, \( R \) the tube radius and \( \nu \) the kinematic viscosity. A parabolic velocity profile is found for \( V_a < 20 \), a transition regime for \( 20 < V_a < 70 \) and plug flow for \( V_a > 70 \). The transition from laminar to turbulent flow is usually expressed in terms of critical Reynolds numbers. Two correlations are given by Ury (1962) and Sergeev (1966), viz.

\[ Re_{crit} = 180 V_a^{2/3} \quad \text{(Ury)} \]

\[ Re_{crit} = 700 V_a^{1/2} \quad \text{(Sergeev).} \]

In our apparatus, in general, the Valensi number is typically 1400, thus indicating that generally plug flow is to be expected. The laminar-turbulent transition takes place at an amplitude of 18 cm (Ury) or 21.6 cm (Sergeev). Thus, under normal experimental conditions, we are operating in the laminar plug flow regime.

One baffle

Typical flow patterns are shown on time sequence photographs in Fig. 11 and schematically in Fig. 12. Figure 11(a) shows the motion momentarily after release of the oscillation. The fluid velocity is low and the flow has not separated at the baffle edge. Figures 11(a)–11(h) show the time evolution of the flow pattern. The flow upstream of the baffle remains laminar at all times. Once a critical velocity is reached the flow separates at the baffle edge. Eddies are formed downstream of the baffle, adjacent to the tube wall. At relatively high amplitudes, as employed here (ca 15 cm) there is “channelling” of the flow through the centre of the baffle, with only the edges separating. At low amplitudes this problem is much reduced. As the
Fig. 9. Photographs of the time evolution of eddies observed for the four right-angled bend section. (a) Primary flow L–R, first oscillation, (b) first flow reversal, (c) primary flow R–L, (d) second flow reversal.
flow develops, the eddies are carried further downstream from the baffle. Figures 11(b) and 11(c) demonstrate clearly the way in which the eddies generated as a result of the separation destroy the laminar boundary layer. This plausibly leads to significant improvement in heat and mass transfer coefficients. Figure 11(d) shows the fluid motion at the point of flow reversal where the upstream flow is momentarily at rest while the deceleration of the fluid on the downstream side is starting to throw the eddies away from the wall and into the main flow, providing good mixing in the process. In Fig. 11(f) the fluid is now moving in the reversal direction. The eddy mixing is continuing on the new upstream side, although the fluid velocity is too low for separation to have occurred on the new downstream side. Figure 11(g) demonstrates that the flow patterns are very rapidly restored after each flow reversal. Whilst separation of a unidirectional flow at a sharp edge will cause the destruction of boundary layers with consequent improvements in transfer coefficients, repeated reversals of the flow are necessary for mixing.

Two baffles

The effect of varying the baffle spacing was investigated in an attempt to find the optimum value. Centre-to-centre baffle spacings of 1, 1 1/2 and 2 pipe diameters (46, 69 and 92 mm) were investigated. Typical time sequence photographs for baffle spacings of 1, 1 1/2 and 2 pipe diameters are shown in Figs 13, 14 and 15, respectively, and schematically in Fig. 16. Interpretation of the time sequence photographs and cine film showed that a spacing of 1 1/2 pipe diameters apparently gave the most effective mixing over the greatest range of amplitudes. This inter-baffle spacing was standardized for all future work.

The pattern of flow development is similar to that found for the single baffle. Early in the cycle the fluid velocity is too low for separation to take place. When a high enough fluid velocity is reached, the flow separates downstream of each baffle, forming bound vortices as seen in Fig. 14(b). The similarity of the flow patterns downstream of each baffle should be noted. This indicates that flow upstream of the baffle has relatively little effect on the flow downstream. This justifies our assumption that the flow patterns seen in Figs 13–15 can be regarded as typical of those found in a system of n baffles.

As in the single baffle case, the reversal of the flow causes the eddies to be thrown away from the wall into the main flow giving rise to excellent eddy mixing patterns. Figure 14(d) shows this mixing to be particularly effective in the interbaffle region. The remaining photographs demonstrate that the flow patterns rapidly re-establish themselves following each flow reversal.
Comparison of Figs 13-15 shows clearly that baffle spacing has an effect on the effectiveness of the mixing. A baffle spacing of 1 pipe diameter seems particularly prone to large scale channeling of the flow through the centre of the baffle with minimal separation and mixing. A spacing of 2 pipe diameters is too great for satisfactory eddy interaction to take place on flow reversal except within a closely defined range of amplitudes. The photographs shown here represent this optimum amplitude. It has been noted that the optimum amplitude of oscillation is between about 3 and 7 cm. Under these conditions, the problem of channeling is minimized.

Energy losses. Figure 17 shows the surface height decay envelope for the cylindrical tube alone (a) and with an increasing number of baffles placed in the tube (b–e). The full oscillatory time evolution curve is shown for the case with nine baffles positioned with a 1½ tube diameter baffle spacing. In each case the starting amplitude was 10 cm and it can be seen that...
Fig. 12. Schematic diagram showing the development of the flow at a single baffle.

Fig. 13. Photographs of the time evolution of the flow past two baffles placed 1 pipe diameter apart. Starting amplitude 5 cm. (a) Primary flow R–L, start of oscillation, flow not yet separated, (b) primary flow R–L, flow separated, (c) primary flow R–L large bound vortex downstream of second baffle, (d) point of flow reversal, (e) flow reversed, (f) primary flow L–R flow pattern re-established, (g) approaching second flow reversal, (h) point of second flow reversal.
dissipation increases with the increasing number of baffles in the tube.

**MODELLING OF ENERGY LOSSES**

*Mathematical formulation*

We follow the method of Mackley and Knott (1980) by applying the time-dependent Bernoulli equation with additional pressure loss terms in order to account for dissipation. The geometry and boundary conditions of the flow are schematically shown in Fig. 18 and the governing second-order non-linear ordinary differential equation is given by

\[
\rho L \frac{d^2x}{dt^2} + F \frac{dx}{dt} + E \frac{dx}{dt} \frac{dx}{dt} = 2 \rho g x = 0. \tag{1}
\]

$L$ is the path length of fluid ($L = 3.056$ m for cylindrical duct), $\rho$ is the fluid density and $g$ the gravitational constant. The term $F \frac{dx}{dt}$ is associated with linear viscous losses in the system and the magnitude of $F$ reflects the magnitude of the viscous damping. Similarly the $E \frac{dx}{dt} \frac{dx}{dt}$ term is associated with the non-linear damping and the magnitude of $E$ corresponds to
the magnitude of the eddy dissipation. If the damping due to terms $F$ and $E$ are zero, eq. (1) has a harmonic form where the angular frequency $\omega$ is given by $\omega = (2g/L)^{1/2}$.

We have solved eq. (1) by two methods, either using a Runge-Kutta recurrence relation or by using an approximate analytic solution given in an appendix to this paper. By adjusting the values of $F$ and $E$ and using a nested iterative search technique we were able to determine the least-squares best fit to an experimental decay envelope. We found that both methods gave close agreement in terms of $E$ and $F$, however for some decay curves we found a range of values within $F$ and $E$ that gave equivalent convergence and this result is partly reflected in the error bars given with values of $E$ and $F$.

**Results**

Values determined for $E$ and $F$ are given, respectively, in Figs 19 and 20 as a function of the starting amplitude and number of baffles present in the cylindrical geometry duct. For each experimental condition the average value of $E$ and $F$ and the standard deviation was determined from ten separate experiments. Comparable data were also obtained for the two-dimensional duct but the results are not reported here.
For the unbaflled tube, $E$ was found to be zero at all amplitudes tested (5, 10 and 15 cm). This confirms our initial visual observations of laminar plug flow. The value of the friction damping parameter is seen to be finite and the amplitude variation is slight. For this situation it is possible to calculate the value of the parameter $F$ theoretically using boundary layer theory. $F$ is given by

$$F = \frac{S}{A} \int \frac{\mu}{\eta} \left[ \frac{\partial u}{\partial r} \right]_{\text{wall}} \frac{dr}{df}$$

where $S$ is the wetted surface area of the tube, $A$ the cross-sectional area of tube and $\mu$ the fluid viscosity. The velocity $u$ is given by Schlichting (1979) from a solution of the Navier–Stokes equation as

$$u = \frac{k}{\omega} \left\{ \sin \omega t - \sqrt{\frac{R}{r}} \exp \left[ -(R-r) \frac{\omega}{\sqrt{2v}} \right] \right\} \times \sin \left[ \omega t - (R-r) \frac{\omega}{2v} \right]$$

where $u$ is the velocity at a distance $R-r$ from the wall, $\omega$ the angular frequency of oscillation, $v$ the kinematic viscosity of the fluid and $k$ a constant given in our case by

$$k = -\omega^2 x_o e^{-\frac{F_0}{2}}$$

$x_o$ is the initial starting amplitude and $\rho$ the fluid density. Theoretical and experimental results are compared in Table 1. Since no account has been taken of end effects in the theoretical case, the agreement is reasonable.

The effect of reducing the length of the centre section was investigated. The standard 1.75 m long section was replaced by one 0.5 m long. As expected, the value of the parameter $F$ was reduced. Table 1 shows that the size of this reduction compares very closely with that predicted theoretically.

The introduction of baffles into the system gives rise to a finite value of the eddy damping coefficient $E$, except in the case where the velocity is insufficiently high to cause the flow to separate and give rise to eddies. Figure 21 shows the variation of $E$ with

---

**Fig. 16.** Schematic diagram showing the development of the flow past two baffles placed 2 pipe diameters apart.

**Fig. 17.** Experimental decay envelopes for the system with various numbers of baffles. (a) No baffles, (b) one baffle, (c) two baffles, (d) five baffles, (e) nine baffles.
Oscillatory flow in ducts containing sharp edges

Fig. 18. Schematic diagram of mathematical model for duct flow.

Fig. 19. Graph of parameter $E$ as a function of starting amplitude and a number of baffles. (■) one baffle, (x) two baffles, (○) three baffles, (▲) five baffles, (□) seven baffles, (△) nine baffles.

Fig. 20. Graph of parameter $F$ as a function of number of baffles and starting amplitude, representative error bars shown for selected points only. (x) 5 cm, (○) 10 cm, (■) 15 cm.

Table 1. Comparison of experimental friction parameter with value calculated from boundary layer theory: cylindrical duct with no baffles

<table>
<thead>
<tr>
<th>Centre section length (m)</th>
<th>Starting amplitude (cm)</th>
<th>Experimental $F$ (kg m$^{-2}$ s$^{-1}$)</th>
<th>Theoretical $F$ (kg m$^{-2}$ s$^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5</td>
<td>5</td>
<td>391</td>
<td>288</td>
</tr>
<tr>
<td>0.5</td>
<td>10</td>
<td>360</td>
<td>288</td>
</tr>
<tr>
<td>0.5</td>
<td>15</td>
<td>320</td>
<td>288</td>
</tr>
<tr>
<td>1.75</td>
<td>5</td>
<td>494</td>
<td>397</td>
</tr>
<tr>
<td>1.75</td>
<td>10</td>
<td>452</td>
<td>397</td>
</tr>
<tr>
<td>1.75</td>
<td>15</td>
<td>462</td>
<td>397</td>
</tr>
</tbody>
</table>

1Calculations based on a total tube length of 1.806 and 3.056 m for the 0.5 and 1.75 m centre sections, respectively.
linear. Amplitude dependence is slight, except for the low amplitude case for up to 5 baffles. It is thought that this may be due to failure of the flow to fully separate and develop, particularly when the amplitude of oscillation reaches a very low level. The eddy damping coefficient may be expressed in terms of velocity heads lost by the expression

\[ \text{no. of velocity heads lost per half cycle} = 2E/\rho. \]

These results are shown in Table 2.

**Energy losses**

The viscous energy loss is given by

\[ \text{viscous energy loss} = \int AFu^2 \, dt \]

and the eddy energy loss by

\[ \text{eddy energy loss} = \int |AEu^3| \, dt. \]

where the integral is evaluated over the first five complete oscillations. Table 2 gives the results of these calculations as well as the value of the peak Reynolds number reached in the apparatus under each set of experimental conditions. Table 2 demonstrates that increasing the number of baffles causes only a small increase in the total energy lost in the system. This is

<table>
<thead>
<tr>
<th>No. of baffles</th>
<th>Starting amplitude (cm)</th>
<th>Peak Re</th>
<th>Energy loss (mJ)</th>
<th>Eddy loss</th>
<th>Friction loss</th>
<th>Velocity heads lost to eddies/baffle/half cycle</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>5</td>
<td>3800</td>
<td>0</td>
<td>35.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>0</td>
<td>137.7</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>0</td>
<td>313.6</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>5</td>
<td>3800</td>
<td>0</td>
<td>37.3</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>38.8</td>
<td>121.1</td>
<td>0.255</td>
<td>1.77</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>126.3</td>
<td>220.3</td>
<td>0.573</td>
<td>2.42</td>
</tr>
<tr>
<td>2</td>
<td>5</td>
<td>3800</td>
<td>2.1</td>
<td>36.2</td>
<td>0.06</td>
<td>0.48</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>48.6</td>
<td>107.8</td>
<td>0.42</td>
<td>1.7</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>138.4</td>
<td>218.4</td>
<td>0.63</td>
<td>1.65</td>
</tr>
<tr>
<td>3</td>
<td>5</td>
<td>3800</td>
<td>7.0</td>
<td>31.8</td>
<td>0.22</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>60.6</td>
<td>98.0</td>
<td>0.62</td>
<td>1.6</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>166.1</td>
<td>196.4</td>
<td>0.85</td>
<td>1.2</td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>3800</td>
<td>10.2</td>
<td>29.7</td>
<td>0.342</td>
<td>1.28</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>84.6</td>
<td>78.3</td>
<td>1.08</td>
<td>1.88</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>225.8</td>
<td>146.7</td>
<td>1.54</td>
<td>1.86</td>
</tr>
<tr>
<td>7</td>
<td>5</td>
<td>3800</td>
<td>16.2</td>
<td>24.1</td>
<td>0.67</td>
<td>1.71</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>96.6</td>
<td>28.5</td>
<td>1.41</td>
<td>1.84</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>257.5</td>
<td>121.9</td>
<td>2.11</td>
<td>1.87</td>
</tr>
<tr>
<td>9</td>
<td>5</td>
<td>3800</td>
<td>22.4</td>
<td>18.2</td>
<td>1.23</td>
<td>2.18</td>
</tr>
<tr>
<td></td>
<td>10</td>
<td>7600</td>
<td>117.4</td>
<td>50.4</td>
<td>2.33</td>
<td>2.14</td>
</tr>
<tr>
<td></td>
<td>15</td>
<td>11,400</td>
<td>293.0</td>
<td>96.6</td>
<td>3.03</td>
<td>2.03</td>
</tr>
</tbody>
</table>
however, a significant redistribution of the energy loss from viscous to eddy losses as shown by the ratio of these figures. Increasing the amplitude of oscillation greatly increases the energy lost by the system. The small size of these energy losses should be noted, all are less than 1 J over a period of approximately 12 s, corresponding to a power loss of approximately 0.1 W.

**DISCUSSION**

We have shown in this paper that oscillatory flow in rectangular ducts containing sharp right-angled bends and oscillatory flow in baffled tubes are both capable of generating flow patterns where complex, large-scale eddy motions are generated in the duct. The time dependence of the motion adds significantly to the richness of the flow pattern complexity. By examining oscillatory flow around a single right-angled bend, or through a baffle within a tube the basic mechanism for the development of eddy formation can be identified. Initially separation must occur downstream of the sharp edge. On flow reversal, fluid then channels between the separated eddy and the wall and causes the initial separated eddy to be ejected into the body of the flow. Each flow reversal generates a new eddy which is subsequently pumped into the body of the flow. Increasing the number of edges or baffles increases the resulting complexity of the flow. Flow patterns where we see this intense eddy mixing effect occur in the range of order $Re_e = 300-100,000$ and $St = 10^{-2}$.

The U-tube manometer arrangement used has enabled us to make estimations of the energy dissipation for each system. In the experimental range tested we have found that the damping was essentially linear when no sharp corners or baffles were present. With the systematic introduction of corners or baffles there was a progressive increase in the non-linear damping term. The frictional losses reflected in the adjustable parameter $F$ were found to agree reasonably well with laminar boundary layer theory for the flow in an un baffled tube. The non-linear losses corresponded to the order of one velocity head per oscillation per baffle and the total energy losses of the system seems remarkably low for the apparent high level of mixing observed from flow visualization.

The geometries we have described are far from complete both in terms of their scale and shape; however in our opinion they give representative information on a range of conditions where efficient mixing can be achieved in ducted flows and this should give guidance for further geometry development and utilization of the effect we have described.

**NOTATION**

$A$ cross section area of duct
$\phi$ parameter in Kryloff–Bogoliuboff equation, $\phi = \tan^{-1} \omega \frac{Q}{F \omega^2}$
$B$ parameter in Kryloff–Bogoliuboff equation, $B = \frac{3\pi E}{8\omega E}$
$E$ eddy damping term, $P_e = E \frac{dx}{dt}$
$F$ viscous damping term, $P_f = F \frac{dx}{dt}$
$L$ path length of fluid in duct
$K$ Keulegan–Carpenter number, $K = \frac{U_o T}{D}$
$P$ fluid density
$Q$ parameter in Kryloff–Bogoliuboff equation, $Q = \frac{F \omega^2}{4\rho g}$
$Re_e$ Reynolds number, $Re_e = \frac{U_o H}{\nu}$
$St$ Strouhal number, $St = \frac{\omega H}{4\pi U_o}$
$T$ period of oscillation
$U_e$ peak channel velocity
$U_o$ peak velocity of oscillatory flow
$X_0$ vertical displacement from equilibrium level in duct
$\omega$ angular frequency of oscillation
$\nu$ kinematic viscosity
$
\mu$ fluid viscosity
$\nu$ fluid kinematic viscosity
$\rho$ fluid density
$\omega$ angular frequency of oscillation
$\phi$ parameter in Kryloff–Bogoliuboff equation, $\phi = \tan^{-1} \omega \frac{Q}{F \omega^2}$

**REFERENCES**


APPENDIX: KRYLOFF–BOGOLIUBOFF SOLUTION OF NON-LINEAR DIFFERENTIAL EQUATIONS

Flower and Aljaff (1980) demonstrated the application of the method devised by Kryloff and Bogoliuboff (KB) for finding an approximate analytic solution to a non-linear differential equation of the form found in this case.

The KB method provides approximate solutions to equations of the form

$$\frac{d^2x}{dt^2} + \mu f\left(x, \frac{dx}{dt}\right) + \omega^2 x = 0$$  \hspace{1cm} (A1)

where $\mu$ is a small constant.

For $\mu = 0$, eq. (A1) is that for simple harmonic motion with a solution of the form

$$x = a \sin (\omega t + \phi)$$  \hspace{1cm} (A2)

$$\frac{dx}{dt} = \omega a \cos (\omega t + \phi).$$  \hspace{1cm} (A3)

The KB method seeks a solution of eq. (A1) of the form of eqs (A2) and (A3) by allowing $a$ and $\phi$ to be functions of time.

Following the method of Flower and Aljaff (1980) and applying our boundary conditions appropriate to our problem we arrive at the following equation for the displacement $x$ as a function of time.

$$x = \left[ \frac{B}{\left( \frac{B}{x_0} + 1 \right) e^{\omega t} - 1} \right] \cos (\omega t + \phi_0 - \pi/2)$$  \hspace{1cm} (A4)

where $\omega = (2g/L)^{1/2}$, $\tan \phi_0 = \omega_0/Q$, $Q = F\omega^2/(4\rho g)$, and $B = 3\pi F/(8\rho E)$. 

